

# Resonance Frequencies of Ventilated Wind Tunnels

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## Abstract

EXPERIMENTS suggest that the theory widely used to predict the transverse resonance frequencies in slotted tunnels is in error in the 0-0.5 Mach number range. One reason for the error is that the theory is based on an unrepresentative wall boundary condition. Moreover, the theory implies that the plenum chamber depth is generally less than twice the tunnel height. An improved theory is developed which shows that the resonance frequencies of ventilated tunnels are influenced by the depth of the plenum chamber for Mach numbers up to about  $M=0.6$ . Although the theory is approximate, it agrees well with experiments for slotted and perforated walls (with both normal and 60 deg inclined holes) in a small pilot wind tunnel (100 × 100 mm). The earlier theory was only valid for slotted working sections. The results are consistent with other experiments, which show that plenum chamber design can influence the flow unsteadiness within the working section of a ventilated tunnel.

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The previous theory for the resonance frequencies<sup>1</sup> assumed that the oscillatory pressure difference across the equivalent homogeneous wall (which replaced the slotted wall) was independent of the plenum chamber size and proportional to the streamline curvature.

The new theory<sup>2</sup> includes a plenum chamber and uses the notation shown in Fig. 1. The two-dimensional working section extends from  $x = -\infty$  to  $+\infty$  and has a uniform flow velocity  $U$  at a Mach number  $M$ . It is surrounded by two plenum chambers, each of depth  $dH/2$ , with zero mean flow. The working section is separated from the plenum chambers by homogeneous ventilated walls, which are thin, rigid, and have no boundary layers. For simplicity, we assume that the mean pressure and static temperature are the same in the working section and plenum chambers. Hence, the densities in the freestream and plenum chambers are the same. We know from the measurements of Smith and Shaw,<sup>3</sup> however, that the static temperature within a cavity is close to the freestream total temperature, not the freestream static temperature; but the error in density is trivial at Mach numbers up to  $M=1.0$ .

For the oscillatory flow we seek compatible solutions for the velocity potentials  $\phi$  and  $\psi$  in the freestream and plenum chambers. The boundary condition on the outer walls of the plenum chamber is simply that the normal velocity should be zero. Thus,

$$\psi_z = 0 \text{ on } y = \pm H(1+d)/2 \quad (1)$$

The specification of boundary conditions on the inner walls of the plenum chamber is controversial. We assume that there is continuity of mass flow from the working section to the

plenum chamber, so that

$$\phi_z = \psi_z \text{ on } y = \pm H/2 \quad (2)$$

In addition, different expressions are developed for the pressure drop across perforated and slotted walls.

For perforated walls of thickness  $Z$ , we note that with a single normal hole of diameter  $D$  and air at rest, a phase lag of 90 deg between the applied pressure and normal velocity is both predicted and measured.<sup>4</sup> In accordance with a recent theoretical study<sup>5</sup> and measurements,<sup>6</sup> we assume that the freestream flow does not alter this relationship and that the hole resistance remains much smaller than the impedance. The pressure drop across a single hole is then generalized to predict the pressure drop across perforated walls at  $z = \pm H/2$  containing many holes to give

$$\phi_x + (\phi_t - \psi_t)/U \pm (ik\omega T/U)\phi_z = 0 \quad (3)$$

where  $k = (1 - \sigma)/\sigma$  is an empirical function of the open-area ratio  $\sigma$ ,  $\omega$  is the circular frequency of the oscillations, and the effective hole diameter is

$$T = 0.85D + Z \quad (4)$$

(This expression may be generalized for inclined holes, Ref. 2, Eq. (20).)

For slotted walls we assume that the steady flow value of  $F$  is still valid for the oscillatory flow and, hence, derive the wall condition

$$\phi_x + (\phi_t - \psi_t)/U \pm (FH/2)\phi_{xz} = 0 \quad (5)$$

where the slot parameter  $F = (2a/\pi H) \ln \csc(b/2a)$ , and  $a$ ,  $b$ , and  $H$  are slot spacing, slot width, and tunnel height,<sup>7</sup> respectively.

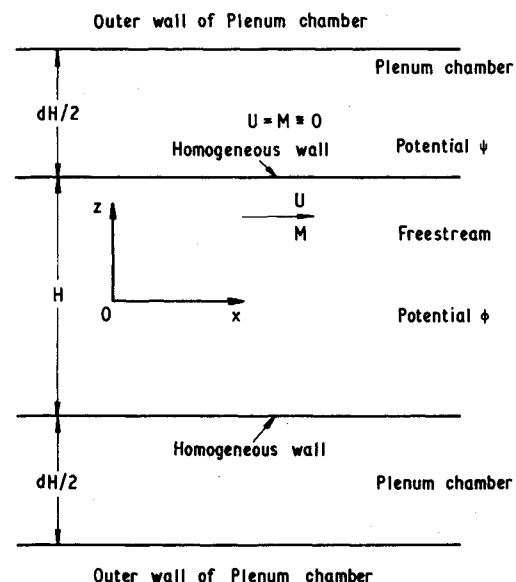


Fig. 1 Notation for calculation of resonance frequencies.

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Table 1 Equations for eigenvalues<sup>a</sup>

Mach number range	Perforated walls [see Eq. (4) for $kl$ ]	Slotted walls [see Eq. (5) for $F$ ]
0-0.618	$\tan p - (\beta^2/R) \cot(Rdp) + (2\beta^2 kT/H)p = 0$ where $R = [(1-M^2) - M^2/(1-M^2)]^{1/2}$	$\tan p - (\beta^2/R) \cos(Rdp) + M^2 Fp = 0$
0.618-1.0	$\tan p + (\beta^2/Q) \coth(Qdp) + (2\beta^2 kT/H)p = 0$ where $Q^2 = -R^2$	$\tan p + (\beta^2/Q) \coth(Qdp) + M^2 Fp = 0$

<sup>a</sup>Wave number  $p = \pi fH/a\beta$ , where  $f$  is the frequency (Hz),  $a$  is the local velocity of sound, and  $\beta = (1-M^2)^{1/2}$ .

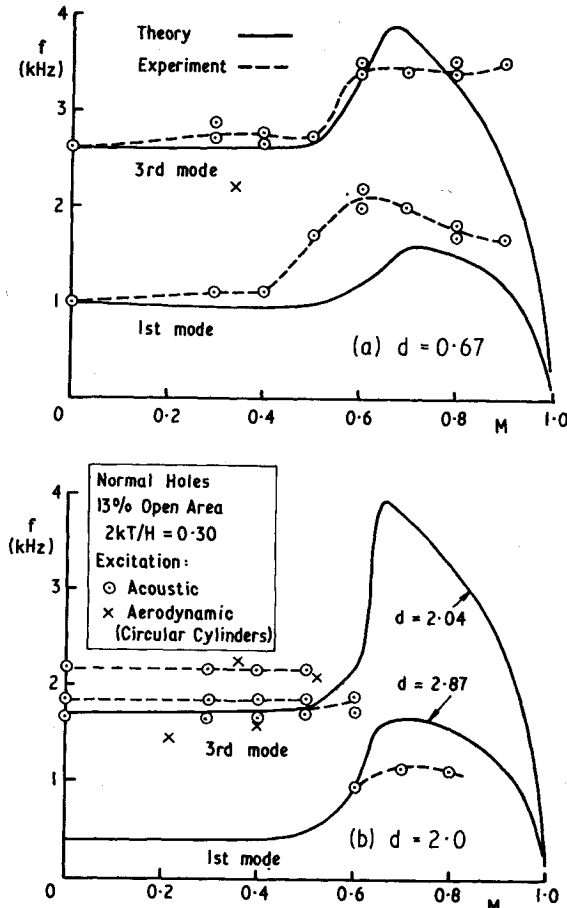


Fig. 2 Perforated walls, resonance frequencies vs Mach number.

Although Eqs. (3) and (5) look quite different, they predict similar trends for the influence of plenum chamber depth, wall porosity, and Mach number on the resonance frequencies. Hence, they may be regarded as different versions of Bernoulli's equation for the oscillatory flow through an orifice or a slot (Ref. 8, Eq. 11.3.37).

With these boundary conditions, eigenvalues  $p$  can be calculated (see Table 1).<sup>2</sup> At low speeds ( $0 < M < 0.5$ ), the resonance frequencies in perforated and slotted tunnels are roughly constant and remain slightly below the closed/closed organ-pipe frequencies at  $M=0$  with a tunnel of height  $H(1+d)$ . From  $M=0.5$  to  $0.7$  the resonance frequencies increase rapidly and the influence of the plenum chamber

becomes negligible. At a special Mach number,  $M = (\sqrt{5} - 1)/2 = 0.618$ , the form of the solution changes suddenly. At this speed, the resonance frequencies of perforated and slotted tunnels are identical, independent of the plenum chamber depth and the wall porosity and equal to the closed-tunnel values ( $p = 0.5\pi$ ). From  $M=0.7$  to  $1.0$ , the resonance frequencies fall monotonically and are virtually unaffected by the depth of the plenum chamber. These predictions were unexpected, but they are broadly confirmed by the experiments<sup>2</sup>; see typical measurements for a perforated tunnel in Fig. 2.

Acoustic theory may be tentatively invoked to explain the sudden change of the solutions at  $M=0.618$ , although the argument is not rigorous.<sup>2</sup> The critical speed corresponds with the condition when the acoustic ray from the model (which would excite resonance in a closed tunnel) strikes a slot and is refracted along the boundary between the mainstream and the plenum chamber, according to the laws given in Ref. 8, p. 705. At this speed, the resonance frequencies would not be influenced by the open-area ratio nor by the depth of the plenum chamber.

This argument rests on the assumption of the travelling wave along the equivalent homogeneous wall. The concept of a continuously varying boundary displacement is easier to accept for a narrow slot than for a succession of individual perforations. Nevertheless, it provides an interesting physical explanation of what the theory predicts and the experiments confirm in both slotted and perforated working sections.<sup>2</sup>

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